

Math 111  
Final Exam

Name KEY  
Section \_\_\_\_\_ Date \_\_\_\_\_

To receive full credit, show your work.

1. Find the equation of the line passing through the point (3, -2), and parallel to the line  $y = -3x + 5$ .

$$y = -3x + 5 \quad y + 2 = -3(x - 3)$$

$$m = -3 \quad y = -3x + 9 - 2$$

$$y = -3x + 7$$

- (a)  $y = -3x + 7$    b.  $y = -3x - 11$    c.  $y = -3x + 11$    d.  $y = -3x + 8$

1. a  
2 points

2.  $f(x) = 2x^2 - x + 5$ . Find  $f(x-1)$ .

$$f(x-1) = 2(x-1)^2 - (x-1) + 5$$

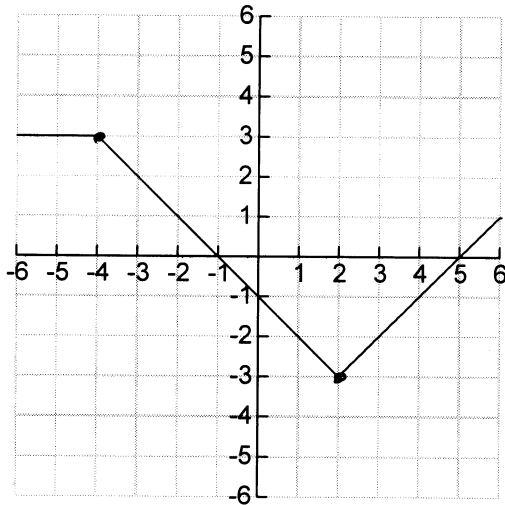
$$= 2(x^2 - 2x + 1) - x + 1 + 5$$

$$= 2x^2 - 4x + 2 - x + 6 = 2x^2 - 5x + 8$$

- a.  $2x^2 - 5x + 7$    b.  $2x^2 - x + 4$    c.  $2x^2 - 5x + 6$    (d)  $2x^2 - 5x + 8$

2. d  
2 points

3. Refer to the graph of  $f(x)$  below. Find  $f(-4) - f(2)$



$$3 - (-3)$$

$$6$$

- a. 1   b. 2   c. 5   (d) 6

3. d  
2 points

4. Use the quadratic formula to solve the equation,  $x^2 + 2x = -10$ . The solutions are:

$$a = 1 \quad b = 2 \quad c = 10$$

$$x = \frac{-2 \pm \sqrt{4 - 4(10)}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$$

- a.  $3+i, 3-i$    (b)  $-1+3i, -1-3i$    c.  $3+2i, 3-2i$    d.  $1+3i, 1-3i$

4. b  
2 points

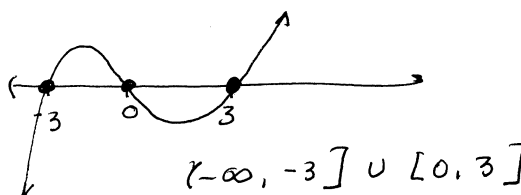
5. A rock falls from an airplane 576 ft. high. As it falls, its height,  $h$ , is given by the formula,  $h = 576 - 16t^2$  where  $t$  represents time in seconds. When the rock hits the ground the height equals 0. How many seconds does it take to hit the ground?

$$\begin{aligned} 576 - 16t^2 &= 0 \\ -16t^2 &= -576 \\ t^2 &= 36 \\ t &= 6 \end{aligned}$$

- a. 4      b. 5      c. 6      d. 7      5.     c      
2 points

6. Solve the inequality  $x^3 - 9x \leq 0$ . Express the answer in interval notation.

$$\begin{aligned} x^3 - 9x &= 0 \\ x(x^2 - 9) &= 0 \\ x(x-3)(x+3) &= 0 \\ x &= 0, \pm 3 \end{aligned}$$



- a.  $[-3, 0] \cup [0, 3]$       b.  $(-\infty, -3] \cup [0, 3]$       c.  $(-\infty, 0] \cup [-3, \infty)$       d.  $[-3, 0] \cup [3, \infty)$

6.     b      
2 points

7. Solve the system of equations for  $x$  and  $y$ .  $y = x^2 - 5x + 10$       The sum of  $x$  and  $y$  is:  
 $3x - y = 6$

$$\begin{aligned} 3x - 6 &= x^2 - 5x + 10 \\ 0 &= x^2 - 8x + 16 \\ 0 &= (x-4)^2 \\ x &= 4 \quad y = 6 \end{aligned}$$

$$\begin{aligned} -y &= -3x + 6 \\ y &= 3x - 6 \end{aligned}$$

- a. 7      b. 8      c. 9      d. 10      7.     d      
2 points

8. The profit for a company's first 5 years of business is shown in the table. Use a graphing calculator to fit the data with a linear function,  $y = ax + b$ . Using the linear function, predict the profit in year 7.

Year ( $x$ )	1	2	3	4	5
Profit, in thousands, ( $y$ )	27	30	36	41	45

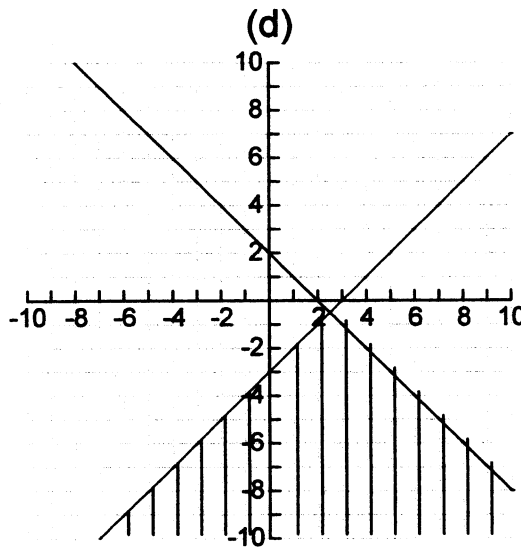
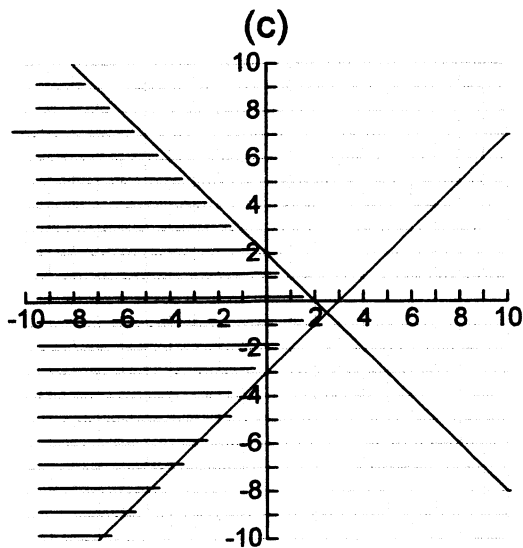
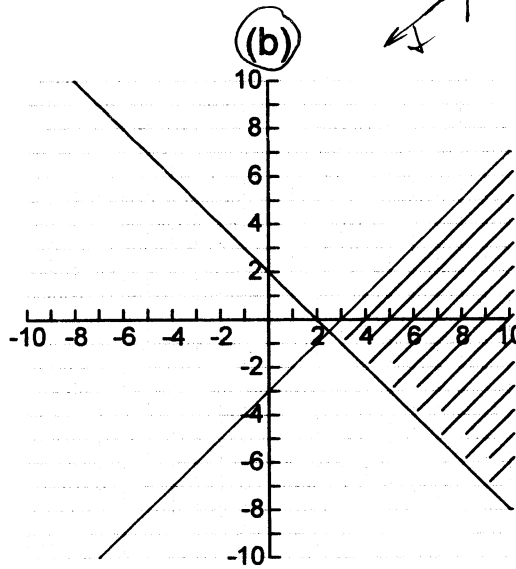
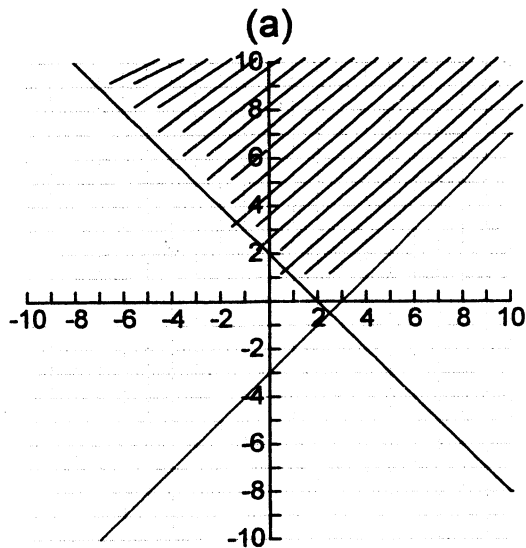
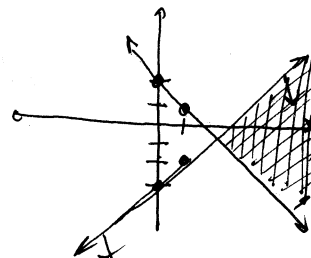
- a. linear function      profit, in thousands      linear function      profit, in thousands  
 a.  $y = 4.7x + 21.7$       54.6      c.  $y = 4.7x + 20.5$       53.4  
 b.  $y = 5.1x + 20.3$       56.0      d.  $y = 5.2x + 21.3$       57.7

8.     a      
2 points

9. Graph the solution of the system of inequalities.

$$y \leq x - 3$$

$$y \geq -x + 2$$



9.     b      
2 points

10. The population of Kenya has an exponential growth rate of 2.1% and an initial population of 24 million people. Using the exponential growth function,  $P(t) = P_0 e^{kt}$ , find  $t$ , the number of years it takes the population to double. Round to the nearest whole year.

$$2 = e^{.021(t)}$$

$$\ln 2 = .021t$$

$$33 = t$$

a. 28 years    b. 31 years    **c. 33 years**    d. 35 years

10.     c      
2 points

11. Factor Completely.  $4n^3 + 108$

$$4(n^3 + 27)$$

$$4(n+3)(n^2 - 3n + 9)$$

11.  $\frac{4(n+3)(n^2-3n+9)}{2 \text{ points}}$

12. Write in simplest form with only positive exponents.

$$\left(\frac{x^{-4}y^3}{3x^{-5}y^8}\right)^3$$

$$\left(\frac{x}{3y^5}\right)^3 = \frac{x^3}{27y^{15}}$$

12.  $\frac{x^3}{27y^{15}}$   
3 points

13. Simplify.  $\frac{x}{x^2+9x+20} - \frac{4}{x^2+7x+12}$

$$\frac{x(x+3) - 4(x+5)}{(x+3)(x+4)(x+5)}$$

$$\frac{x^2 + 3x - 4x - 20}{LCD} = \frac{(x-5)(x+4)}{LCD} = \frac{x-5}{(x+5)(x+3)}$$

13.  $\frac{x-5}{(x+5)(x+3)}$   
3 points

14. Given two points A(4, -6) and B(1, 0).

a) Find the distance between A and B. Give exact value.

$$d = \sqrt{(4-1)^2 + (-6-0)^2}$$

$$= \sqrt{9 + 36} = \sqrt{45}$$

$$= 3\sqrt{5}$$

a.  $\frac{d = 3\sqrt{5}}{2 \text{ points}}$

b) Find the slope and write the equation of the line AB in standard form.

$$m = \frac{-6}{4-1} = \frac{-6}{3} = -2$$

$$y - 0 = -2(x - 1)$$

$$y = -2x + 2$$

$$y = -2x + 2$$

b.  $\frac{2x + y = 2}{3 \text{ points}}$

15. For  $f(x) = x^2 + 10$  and  $g(x) = \sqrt{x-1}$ , find

a.  $f \circ g = f(\sqrt{x-1}) = x - 1 + 10$   
 $= x + 9$

a.  $\frac{x + 9}{2 \text{ points}}$

b.  $g(f(0)) = g(10) = \sqrt{9} = 3$

b.  $\frac{3}{1 \text{ point}}$

16. Solve the equation  $\frac{x}{x+4} = \frac{3}{x-1}$   $x \neq -4, 1$

$$x(x-1) = 3(x+4)$$

$$x^2 - x - 3x - 12 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6, -2$$

6.  $\underline{\{6, -2\}}$   
3 points

17. Solve the inequality, writing your answer in interval notation.

$$-3 \leq 2(x+3) < 6$$

$$-3 \leq 2x+6 < 6$$

$$-9 \leq 2x \leq 0$$

$$-\frac{9}{2} \leq x \leq 0$$

7.  $\underline{[-\frac{9}{2}, 0)}$   
3 points

18. Find the following for  $f(x) = 2x^2 - x - 36$ .

a. x-intercepts  $2x^2 - x - 36 = 0$   
 $(2x-9)(x+4) = 0$

a.  $\underline{(\frac{9}{2}, 0), (-4, 0)}$   
2 points

b. y-intercept

b.  $\underline{(0, -36)}$   
1 point

c. vertex  $h = -\frac{b}{2a} = \frac{1}{4}$

$$k = f(\frac{1}{4}) = 2(\frac{1}{16}) - \frac{1}{4} - 36$$

$$= -\frac{289}{8} = -36\frac{1}{8}$$

c.  $\underline{(\frac{1}{4}, -289/8)}$   
2 points

d. line of symmetry

d.  $\underline{x = 1/4}$   
1 point

19. Perform the following operations:

a.  $(3-4i) + (4+2i)$

a.  $\underline{7-2i}$   
1 point

b.  $(3-4i) - (4+2i)$

b.  $\underline{-1-6i}$   
1 point

c.  $(3-4i)(4+2i) = 12 + 6i - 16i - 8i^2$   
 $= 12 - 10i + 8$

c.  $\underline{20-10i}$   
2 points

d.  $\frac{(3-4i)(4-2i)}{(4+2i)(4-2i)} = \frac{12-6i-16i+8i^2}{16-4i^2}$   
 $= \frac{4-22i}{20}$

d.  $\underline{\frac{2}{10} - \frac{11}{10}i}$   
2 points

$$= \frac{2}{10} - \frac{11}{10}i$$

20. Consider the polynomial function  $P(x) = (x+1)^3(x+2)(x-2)^2$ .

a. Determine the degree of the polynomial.

a.  $\frac{6^{th}}{1 \text{ point}}$

b. Determine the zeros of the function and state the multiplicity

3 points

ZERO VALUES

MULTIPLICITY

$$x = -1$$

$$n = 3$$

$$x = -2$$

$$n = 1$$

$$x = 2$$

$$n = 2$$

c. Determine the end behavior of the graph of the function.



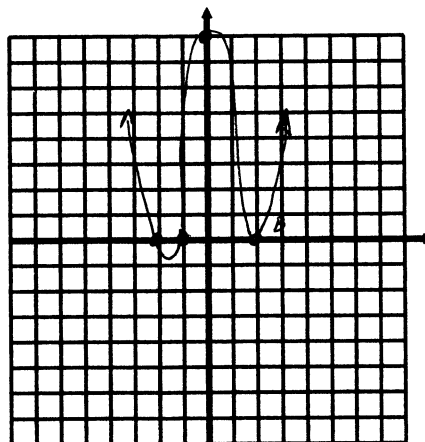
c.  $\frac{\uparrow \uparrow}{2 \text{ points}}$

d. Determine the y-intercept of the function.

$$P(0) = 2(-2)^2 = 8$$

d.  $\frac{(0, 8)}{1 \text{ point}}$

e. Sketch the graph of the function. 3 points



21. Given that 1 and 3 are zeros of the function

$P(x) = x^4 - 4x^3 + 7x^2 - 16x + 12$ , find the remaining zeros.

Show all work to receive credit.

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 7 & -16 & 12 \\ & & 1 & -3 & 4 & 12 \\ \hline 3 & 1 & -3 & 4 & -12 & 0 \\ & & 3 & 0 & 12 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$$x^2 = -4$$

$$x = \pm 2i$$

21.  $\frac{x = \pm 2i}{3 \text{ points}}$

22. Consider the function:  $f(x) = \frac{(3x-1)(x+2)}{x^2+x-12}$   
 $(x+4)(x-3)$

- a. Give the equation(s) of any vertical, horizontal, or oblique asymptotes of the function.

Vertical asymptotes:  $x = -4, x = 3$   
2 points

Horizontal asymptote:  $y = 3$   
1 point

Oblique asymptote:  $none$   
1 point

- b. Determine the coordinates of the x and y-intercepts

X-intercept(s):  $(\frac{1}{3}, 0), (-2, 0)$   
2 points

Y-intercept:  $(0, \frac{1}{6})$   
1 point

23. a. How would you obtain the graph of  $f(x) = \log\left(x - \frac{5}{2}\right) - 1$  from the graph of  $f(x) = \log x$  ?

horiz shift to the right  $\frac{5}{2}$ , vert shift down 1  
2 points

- b. What is the domain of the function in part a? (Give the answer in set builder notation)

$x - \frac{5}{2} > 0$   
 $x > \frac{5}{2}$

$\{x \mid x > \frac{5}{2}\}$   
b.  $(\frac{5}{2}, \infty)$   
2 points

24. Suppose \$4500 is invested at 3.5%, compounded monthly. How much will be in the account after 3 years?  $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 4500$   
 $r = .035$   
 $n = 12$   
 $t = 3$

$A = 4500\left(1 + \frac{.035}{12}\right)^{12 \cdot 3}$

24.  $\$4997.43$   
3 points

25. Simplify using the rules of logarithms (must show steps and do not use calculator):

$$5\log_4 2 + \log_4 2$$

$$\begin{aligned} \log_4 2^5 + \log_4 2 \\ \log_4 2^{5 \cdot 2} &= \log_4 2^6 \\ &= \log_4 64 = 3 \end{aligned}$$

25.  $\frac{3}{3 \text{ points}}$

26. Solve:  $2\log_3 x - \log_3(x-2) = 2$

$$\log_3 x^2 - \log_3(x-2) = 2$$

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

$$9 = \frac{x^2}{x-2}$$

$$x^2 = 9x - 18$$

$$\begin{aligned} x^2 - 9x + 18 &= 0 \\ (x-6)(x-3) &= 0 \\ x &= 6, 3 \end{aligned}$$

26.  $\frac{x = 3, 6}{3 \text{ points}}$

27. Solve analytically:  $2x + 3y = -1$   
 $3x - y = 4$

$$-y = -3x + 4$$

$$y = 3x - 4$$

$$2x + 9x - 12 = -1$$

$$11x = 11$$

$$x = 1$$

$$\begin{aligned} 3 - y &= 4 \\ -y &= 1 \\ y &= -1 \end{aligned}$$

27.  $\frac{(1, -1)}{3 \text{ points}}$

28. Write a system of equations and solve the following application. Toby split his savings into two different investments, one earning 5% and the other earning 7%. He put twice as much in the investment earning 7%. In one year he earned \$475 in interest. How much money did he invest in each account?

$$\begin{aligned} x &= \text{amt invested @ } 5\% \\ y &= \text{ " " " } 7\% \end{aligned}$$

$$2x = y$$

$$.05x + .07y = 475$$

$$.05x + .07(2x) = 475$$

$$.05x + .14x = 475$$

$$.19x = 475$$

$$x = 2500$$

$$y = 5000$$

28.  $\frac{\$2500 @ 5\%}{\$5000 @ 7\%}$   
4 points



29. Solve the system of equations using matrix methods. Write the matrix equation  $AX = B$  and give solution in the form  $(x, y, z)$ .

$$2x - 2y + 3z = -1$$

$$2x - 6y - 4z = 9$$

$$x + y + z = -6$$

- a. Write the matrix equation,  $AX = B$ .

$$\begin{bmatrix} 2 & -2 & 3 \\ 2 & -6 & -4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ -6 \end{bmatrix}$$

a. \_\_\_\_\_  
2 points

- b. Find  $A^{-1}$ , fractions only.

$$A^{-1} = \begin{bmatrix} -\frac{1}{16} & \frac{5}{32} & \frac{13}{16} \\ -\frac{3}{16} & -\frac{1}{32} & \frac{7}{16} \\ \frac{1}{4} & -\frac{1}{8} & -\frac{1}{4} \end{bmatrix}$$

b. \_\_\_\_\_  
2 points

- c. Give solution in ordered triple, fractions only.

$$\left( -\frac{109}{32}, \frac{-87}{32}, \frac{1}{8} \right)$$

c. \_\_\_\_\_  
2 points

#### Notice

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade (D or better) is required to take Math 118 or for this course to satisfy the  $A_2$ , Mathematics component of the University Core Curriculum.