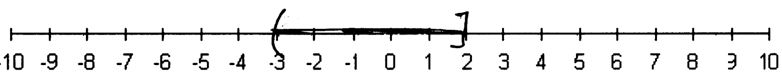
Math 111
Final Exam Name KEY
Section Date

1. (a) Graph the solution set specified by the given statement. **2 points**
$\{x \mid x > -3 \text{ and } x \le 2\}$



(b) Express the final solution set from part (a) in **simplest interval format**.

Interval(s): $(-3, 2]$

2 points

2. Write in simplest form with only **positive exponents.**

$$\left(2a^{\frac{2}{3}}b^{-4}\right)^3\left(-3a^{-\frac{1}{2}}b^{\frac{3}{2}}\right)^2$$

$(8a^2b^{-12})(9a^{-1}b^3)$

$72ab^{-9}$

$\frac{72a}{b^9}$

2. $\frac{72a}{b^9}$

4 points

3. Perform the indicated operations and simplify where possible. **Show all work** to receive credit.

$$\frac{3y}{y^2-7y+10}-\frac{2y}{y^2-8y+15}$$

$\frac{3y}{(y-2)(y-5)} - \frac{2y}{(y-3)(y-5)}$

$\frac{3y(y-3)-2y(y-2)}{(y-2)(y-3)(y-5)}$

$\frac{3y^2-9y-2y^2+4y}{(y-2)(y-3)(y-5)}$

$\frac{y^2-5y}{(y-2)(y-3)(y-5)} = \frac{y\cancel{(y-5)}}{(y-2)(y-3)\cancel{(y-5)}}$

3. $\frac{y}{(y-2)(y-3)}$ or $\frac{y}{y^2-5y+6}$

3 points

4. Write an equation for a function that has the shape of $y=|x|$, but is shifted left 4 units and up 3 units.

4. $y=|x+4|+3$

2 points

5. The points (-2, 3) and (4, -5) are the endpoints of the diameter of a circle. Find the length of the **radius** of the circle.

$d = \sqrt{(-2-4)^2 + (3--5)^2}$

$d = \sqrt{(-6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100}$

$d = 10$

$r = \frac{10}{2} = 5$

5. 5

3 points

6. Find the equations of the line that passes through the point (-1, 5) and is perpendicular to the line $3x - 4y = -8$. Write in **slope-intercept form**.

$3x - 4y = -8$

$-4y = -3x - 8$

$y = \frac{3}{4}x + 2$

$m = \frac{3}{4}$

$(-1, 5)$ $m = \frac{-4}{3}$

$y - 5 = \frac{-4}{3}(x+1)$

$3y - 15 = -4x - 4$

$3y = -4x + 11$

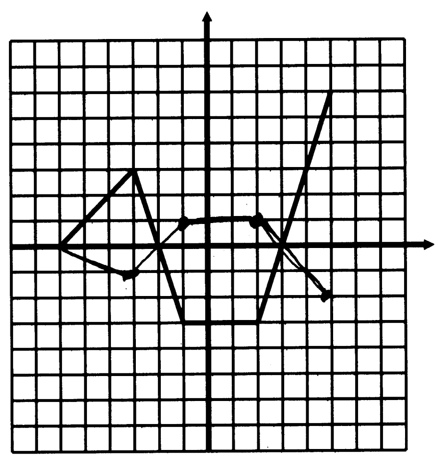
$y = \frac{-4}{3}x + \frac{11}{3}$

6. $y = \frac{-4}{3}x + \frac{11}{3}$

4 points

7. A graph of $y = f(x)$ is given below. No formula for f is given.

Graph $y = -\frac{1}{3}f(x)$.



4 points

8. Solve the inequality $|y+5| \geq 2$ giving the solution in interval notation.

$y + 5 \geq 2$ or $y + 5 \leq -2$

$y \geq -3$ or $y \leq -7$

8. $(-\infty, -7] \cup [-3, \infty)$

3 points

9. Solve for a: $\dfrac{5}{a-4}-\dfrac{3}{a-1}=\dfrac{a+1}{a-4}$

$5(a-1)-3(a-4)=(a-1)(a+1)$

$5a-5-3a+12=a^2-1$

$2a+7=a^2-1$

$a^2-2a-8=0$

$(a-4)(a+2)=0$

$a=4 \quad a=-2$

$a\neq 4$

9. $\{-2\}$

3 points

10. Consider the function $g(x)=x^2-15x+36$ and find:

a) the zeros of $g(x)$

$(x-12)(x-3)=0$

$x=12 \quad x=3$

a) 12, 3

2 points

b) the vertex of the graph of $g(x)$

$x=\frac{-b}{2a}=\frac{15}{2}$

$y=g(\frac{15}{2})=(\frac{15}{2})^2-15(\frac{15}{2})+36=\frac{-81}{4}$

b) $(7.5, -20.25)$

2 points

c) the range of $g(x)$ (in interval notation): c) $[-20.25, \infty)$

2 points

11. Solve for b: $-2(b+3)^2=40$

$(b+3)^2=-20$

$b+3=\pm\sqrt{-20}$

$b+3=\pm 2i\sqrt{5}$

$b=-3\pm 2i\sqrt{5}$

11. $-3\pm 2i\sqrt{5}$

3 points

12. A ball is thrown vertically upward with an initial speed of 48 ft/s. Its height, in feet, after t seconds is given by $h(t) = -16t^2 + 48t$. Find the **maximum height** of the ball and the **time** when it reaches that height.

$\frac{-b}{2a} = \frac{-48}{-32} = 1.5$

$h(1.5) = -16(1.5)^2 + 48(1.5) = 36$

time = 1.5 sec

12. height = 36 ft

4 points

13. Use division to find the quotient $Q(x)$ and the remainder $R(x)$, and express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$.

$P(x) = x^3 - 2x^2 + x - 6$

$d(x) = x + 3$

$$\begin{array}{r} x^2 - 5x + 16 \\ x+3 \overline{) x^3 - 2x^2 + x - 6} \\ x^3 + 3x^2 \\ \hline -5x^2 + x \\ -5x^2 - 15x \\ \hline 16x - 6 \\ 16x + 48 \\ \hline -54 \end{array}$$

13. $P(x) = (x+3)(x^2 - 5x + 16) + {}^{-}54$

3 points

14. Find a polynomial of degree 4 with -2 as a zero of multiplicity 2 and 0 and 3 as zeros of multiplicity 1 and write as $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} \ldots + a_1 x + a_0$.

$P(x) = x(x-3)(x+2)^2$

$P(x) = x(x-3)(x^2+4x+4)$

$P(x) = (x^2-3x)(x^2+4x+4)$

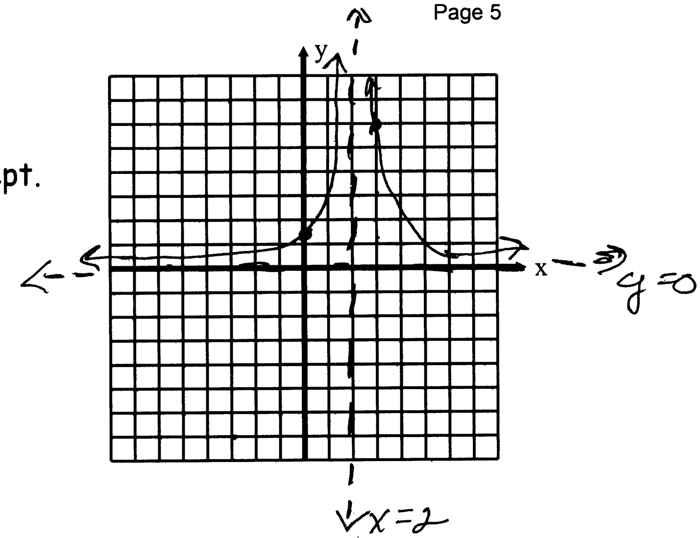
$P(x) = x^4 + 4x^3 + 4x^2 - 3x^3 - 12x^2 - 12x$

$P(x) = x^4 + x^3 - 8x^2 - 12x$

14. $P(x) = x^4 + x^3 - 8x^2 - 12x$

4 points

15. Make a graph of $f(x) = \dfrac{6}{(x-2)^2}$.



(a) <u>Label</u> all the asymptotes and y-intercept.

V.A. $x=2$ y-int $(0, \frac{3}{2})$ **3 points**

H.A. $y=0$

(b) Give the domain in interval notation.

$(-\infty, 2) \cup (2, \infty)$

2 point

(c) Give the $x-$intercepts.

NONE

1 point

16. For the polynomial function $P(x) = x^4 - 2x^3 - 5x^2 + 4x + 6$, solve $P(x) = 0$.

$-1 \rfloor$ 1 −2 −5 4 6
−1 3 2 −6
1 −3 −2 6 $\lfloor 0$

$3 \rfloor$ 1 −3 −2 6
3 0 −6
1 0 −2 $\lfloor 0$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

16. $\{-1, 3, -\sqrt{2}, \sqrt{2}\}$

4 points

17. Solve for x and write in interval form. $3x^2 < 17x - 10$

$3x^2 - 17x + 10 < 0$

$(3x-2)(x-5) < 0$

3x−2: − + +
x−5: − − +
$\frac{2}{3}$ 5

17. $(\frac{2}{3}, 5)$

3 points

18. The table below shows the average yearly income, in dollars, of individuals based on years of schooling. Use the calculator to model the data with a linear function.

Years of Schooling, x	Average Income, y
8	$16,000
10	$19,000
12	$25,000
14	$28,000

(a) $f(x) = 2.1x - 1.1$ in thousands

3 points

(b) Use the function to estimate the average yearly income for an individual with 16 years of schooling.

$f(16) = 32.5$

(b) $32,500

2 points

19. Solve the equations for x:

(a) $4^{3x+1} = 16^{x+1}$

$4^{3x+1} = 4^{2(x+1)}$

$3x+1 = 2x+2$

$x = 1$

(a) $\{1\}$

3 points

(b) $\log_2 x + \log_2(x-8) = 7$

$\log_2 x(x-8) = 7$

$x^2 - 8x = 2^7$

$x^2 - 8x - 128 = 0$

$(x-16)(x+8) = 0$

$x = 16$ or $\cancel{x = -8}$

(b) $\{16\}$

3 points

20. If $3500 is deposited in an account that pays 5.4% APR compounded continuously. How long will it take to double the original deposit? (Round to the nearest tenth of a year) $P = P_0e^{rt}$

$7000 = 3500e^{.054t}$

$2 = e^{.054t}$

$\ln 2 = .054t$

$\frac{\ln 2}{.054} = t \approx 12.8$

20. 12.8 yrs

4 points

21. Graph the function. Show asymptotes and intercepts.

$F(x) = 2^x - 3$

$y = 2^x - 3$

h.a. $y = -3$

Let $x = 0$

$y = 2^0 - 3 = 1 - 3 = -2$ $(0,-2)$

Let $y = 0$

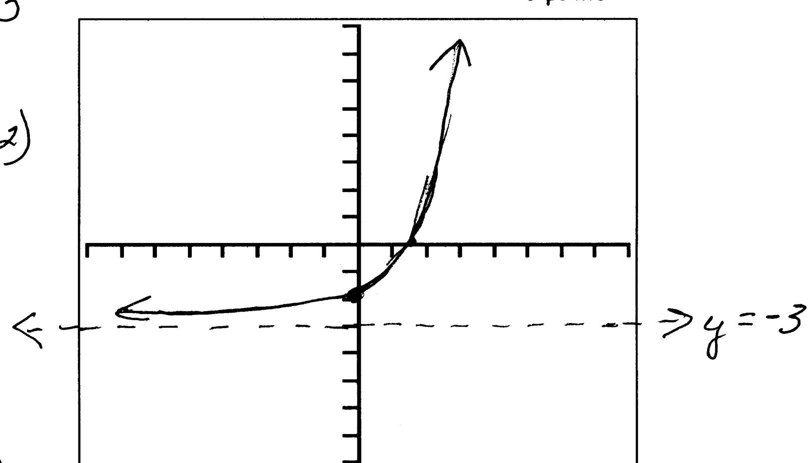
$0 = 2^x - 3$

$3 = 2^x$

$\ln 3 = x \ln 2$

$\frac{\ln 3}{\ln 2} = x \approx 1.58$ $(1.58, 0)$

3 points



Xscl = 1 Yscl = 1

22. Solve for (x, y) using substitution or elimination: $-x + 2y = 2$
$3x + y = 15$

$-3x + 6y = 6$
$3x + y = 15$
$7y = 21$
$y = 3$
$-x + 2(3) = 2$
$-x + 6 = 2$
$-x = -4$
$x = 4$

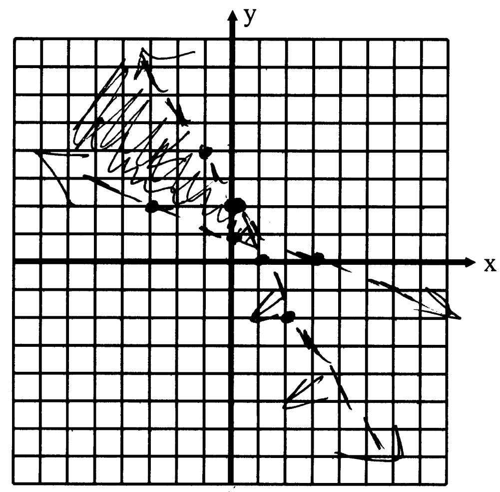
22. $\{(4, 3)\}$

3 points

23. Graph the solution to the system of linear inequalities.

$2x + y < 2$
$x + 3y > 3$

$y < -2x + 2$
$3y > -x + 3$
$y > -\frac{1}{3}x + 1$



4 points

24. Solve for (x, y) algebraically: $x^2 + y^2 = 25$
$2x + y = 10$

$y = -2x + 10$

$x^2 + (-2x + 10)^2 = 25$
$x^2 + 4x^2 - 40x + 100 = 25$
$5x^2 - 40x + 75 = 0$
$x^2 - 8x + 15 = 0$
$(x - 3)(x - 5) = 0$
$x = 3$ $\quad$ $x = 5$
$y = -6 + 10 = 4$ $\quad$ $y = -10 + 10 = 0$
$(3, 4)$ $\quad$ $(5, 0)$

24. $\{(3, 4), (5, 0)\}$

4 points

25. Consider the system:
$$\begin{aligned}4x-y+z&=-5\\2x+2y+3z&=10\\5x-2y+6z&=1\end{aligned}$$

(a) Write a matrix equation equivalent to the system.

(a) $\begin{bmatrix}4 & -1 & 1\\2 & 2 & 3\\5 & -2 & 6\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-5\\10\\1\end{bmatrix}$

2 points

(b) Use your calculator and the inverse of the coefficient matrix of the matrix equation to solve the system.

(b) $\{(-1, 3, 2)\}$

2 points

26. Use a system of equations to solve the following:

A grocer sells oranges for $0.95 each and grapefruit for $1.05 each. You purchase a mix of 16 pieces of fruit and pay $15.90. How many of each type of fruit did you buy?

x = # oranges y = # grapefruit

M_{-95} $x+y=16$

M_{100} $.95x+1.05y=15.90$

$95x+105y=1590$

$-95x-95y=-1520$

$10y=70$

$y=7$

$x+7=16$

$x=9$

26. 9 oranges
7 grapefruit

4 points

Notice

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade (D or better) is required to take Math 118 or for this course to satisfy the A_2, Mathematics component of the University Core Curriculum.